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ROCKET BOOSTER CONTROL

SECTION 12

STATE DETERMINATION FOR A FLEXIBLE VEHICLE WITHOUT A MODE SHAPE REQUIREMENT

NASA Contract NASw-563

MILITARY PRODUCTS GROUP RESEARCH DEPARTMENT

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(2)

OTS

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FOREWORD

This document is one of sixteen sections that comprise the final report prepared by the Minneapolis-Honeywell Regulator Company for the National Aeronautics and Space Administration under contract NASw-563. The report is issued in the following sixteen sections to facilitate updating as progress warrants:

1541-TR 1	Summary
1541-TR 2	Control of Plants Whose Representation Contains Derivatives of the Control Variable
1541-TR 3	Modes of Finite Response Time Control
1541-TR 4	A Sufficient Condition in Optimal Control
1541-TR 5	Time Optimal Control of Linear Recurrence Systems
1541-TR 6	Time-Optimal Bounded Phase Coordinate Control of Linear Recurrence Systems
1541-TR 7	Penalty Functions and Bounded Phase Coordinate Control
1541-TR 8	Linear Programming and Bounded Phase Coordinate Control
1541-TR 9	Time Optimal Control with Amplitude and Rate Limited Controls
1541-TR 10	A Concise Formulation of a Bounded Phase Coordinate Control Problem as a Problem in the Calculus of Variations
1541-TR 11	A Note on System Truncation
1541-TR 12	State Determination for a Flexible Vehicle Without a Mode Shape Requirement
1541-TR 13	An Application of the Quadratic Penalty Function Criterion to the Determination of a Linear Control for a Flexible Vehicle
1541-TR 14	Minimum Disturbance Effects Control of Linear Systems with Linear Controllers
1541-TR 15	An Alternate Derivation and Interpretation of the Drift-Minimum Principle
1541-TR 16	A Minimax Control for a Plant Subjected to a Known Load Disturbance

Section 1 (1541-TR 1) provides the motivation for the study efforts and objectively discusses the significance of the results obtained. The results of inconclusive and/or unsuccessful investigations are presented. Linear programming is reviewed in detail adequate for sections 6, 8, and 16.

It is shown in section 2 that the purely formal procedure for synthesizing an optimum bang-bang controller for a plant whose representation contains derivatives of the control variable yields a correct result.

In section 3 it is shown that the problem of controlling m components $(1 < m \leq n)$, of the state vector for an n-th order linear constant coefficient plant, to zero in finite time can be reformulated as a problem of controlling a single component.

Section 4 shows Pontriagin's Maximum Principle is often a sufficient condition for optimal control of linear plants.

Section 5 develops an algorithm for computing the time optimal control functions for plants represented by linear recurrence equations. Steering may be to convex target sets defined by quadratic forms.

In section 6 it is shown that linear inequality phase constraints can be transformed into similar constraints on the control variables. Methods for finding controls are discussed.

Existence of and approximations to optimal bounded phase coordinate controls by use of penalty functions are discussed in section 7.

In section 8 a maximum principle is proven for time-optimal control with bounded phase constraints. An existence theorem is proven. The problem solution is reduced to linear programming.

A backing-out-of-the-origin procedure for obtaining trajectories for time-optimal control with amplitude and rate limited control variables is presented in section 9.

Section 10 presents a reformulation of a time-optimal bounded phase coordinate problem into a standard calculus of variations problem.

A mathematical method for assessing the approximation of a system by a lower order representation is presented in section 11.

Section 12 presents a method for determination of the state of a flexible vehicle that does not require mode shape information.

The quadratic penalty function criterion is applied in section 13 to develop a linear control law for a flexible rocket booster.

In section 14 a method for feedback control synthesis for minimum load disturbance effects is derived. Examples are presented.

Section 15 shows that a linear fixed gain controller for a linear constant coefficient plant may yield a certain type of invariance to disturbances. Conditions for obtaining such invariance are derived using the concept of complete controllability. The drift minimum condition is obtained as a specific example.

In section 16 linear programming is used to determine a control function that minimizes the effects of a known load disturbance.

STATE DETERMINATION FOR A FLEXIBLE VEHICLE WITHOUT A MODE SHAPE REQUIREMENT*

By E. E. Fisher‡

ABSTRACT

A problem involving state determination for a flexible vehicle without known mode shapes is considered. Conditions are established which allow a least squares estimate of certain system parameters. Knowledge of these parameters allows state determination.

A

INTRODUCTION

The motion of a flexible vehicle is assumed given by the equation

$$\dot{x} = Ax + bu \tag{1}$$

where x is an n-vector representing the state of the system, u is a scalar control variable, A is a constant n by n matrix, and b is a constant n-vector. The system is considered to be controllable, which is taken to mean that the vectors $A^{n-1}b, A^{n-2}, \ldots, Ab, b$ are linearly independent**. A vector z which is related to x by a non-singular transformation

$$z = Mx \tag{2}$$

is observable (capable of being measured). M is a constant non-singular n by n matrix which depends on mode shape data but is unknown. The matrices A and b are assumed known. Time histories of z and u are assumed given. Conditions are sought which allow the determination of the matrix M from the known data.

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* Prepared under contract NASw-563 for the NASA

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** See for instance reference 1

This would allow estimation of x by the equation

$$x = M^{-1}z \tag{3}$$

ANALYSIS

Use is made of equations 1,2, and 3 to yield an equation of motion for z given by

$$\dot{z} = MAM^{-1}z + Mbu. \tag{4}$$

The substitution

$$C \triangleq MAM^{-1},\ d \triangleq Mb \tag{5}$$

reduces equation (4) to

$$\dot{z} = Cz + du. \tag{6}$$

Finally, this is written as the integral equation

$$z(t) - z(t_o) = C\int_{t_o}^{t} z(\tau)d\tau + d\int_{t_o}^{t} u(\tau)d\tau. \tag{7}$$

If the components of the vector $z(t) - z(t_o)$ are denoted by $f_1(t), f_2(t), \ldots, f_n(t)$, the components of the vector $\int_{t_o}^{t} z(\tau)d\tau$ are denoted by $g_1(t), g_2(t), \ldots g_n(t)$, and $\int_{t_o}^{t} u(\tau)d\tau$ is denoted by $v(t)$, then equation 7 takes the component form

$$f_i(t) = c_{i1}g_1(t) + c_{i2}g_2(t) + \ldots + c_{in}g_n(t) + d_iv(t) \tag{8}$$

for $i = 1,2,\ldots,n$

Since the functions $g_1, g_2, \ldots g_n, f_1, \ldots f_n, v$ are known functions or integrals of known functions, they are known.

Suppose that by comparison of the known functions $g_1, g_2, \ldots, g_n, v$ with the functions f_i it were possible to determine the matrices C and d. Then the matrix M would be known because M satisfies the equation

$$L = MN \text{ or } M = LN^{-1} \tag{9}$$

where L is a matrix with columns $d, Cd, C^2d, \ldots C^{n-1}d$; and N is a matrix with columns $b, Ab, A^2b, \ldots, A^{n-1}b$. The matrix N is non-singular because the vectors $b, Ab, A^2b, \ldots, A^{n-1}b$ are linearly independent.

The remainder of the analysis is devoted to establishing a sufficient condition which allows the estimation of the matrices C and d by the method of least squares.

Minimization with respect to c_{ij} and d_i of the n error functions $E_1, E_2, \ldots E_n$ given by

$$E_i = \int_{t_o}^{T} [f_i(t) - \sum_{j=1}^{n} c_{ij}g_j(t) - d_i v(t)]^2 dt \tag{9}$$

for $i = 1,2,\ldots,n$ will yield n sets of n+1 linear equations for the unknowns c_{ij} and d_i. In the event that the functions $g_1, g_2, \ldots g_n$, and v are linearly independent these equations will be soluble for c_{ij} and d_i. A condition which is sufficient to insure this linear independence is now established.

Since the set of functions $g_1, g_2, \ldots g_n, v$ are integrals of the functions $z_1, z_2, \ldots z_n, u$ they are linearly independent if and only if the latter are. The latter set is, however, obtained from the state components and control variable, $x_1, x_2, \ldots x_n, u$ by a non-singular linear transformation. These are linearly independent if and only if the state components and the control variable are linearly independent. It will now be shown that a requirement for non-linearity of the controller is sufficient to insure this linear independence.

A controller $u = u(x)$ will be called non-linear if u is not given by an equation of the form

$$u = k_1x_1 + k_2x_2+\ldots+k_nx_n \tag{9}$$

where $k_1,k_2,\ldots k_n$ are constants and the equation holds identically in time.

The common linear feedback controller is not non-linear. On the other hand, a bang-bang controller is non-linear. This is seen by a continuity argument. The zero function is seen to be a linear controller with $k_1 = k_2 = \ldots = k_u = 0$. The following theorem finishes the discussion.

<u>THEOREM.</u> Let $x_1,x_2,\ldots,x_n$ be state vector components and let $u = u(x)$ be a control variable which together satisfy equation (1). Let the system given by equation (1) be controllable. Then if the controller $u = u(x)$ is a non-linear controller the functions $x_1,x_2,\ldots,x_n,u$ will be linearly independent.

<u>PROOF:</u> The functions $x_1,x_2,\ldots,x_n,u$ will be linearly independent if and only if there is no non-trivial linear relationship between them. Thus it is sufficient to prove that the truth of a linear relationship $e_1x_1 + e_2x_2 +\ldots+ e_nx_n + e_{n+1}u = 0$ necessarily implies that $e_1 = e_2 = \ldots = e_n = e_{n+1} = 0$.

The truth of a linear relationship with e_{n+1} nonzero would imply that u was a linear combination of $x_1,x_2,\ldots,x_n$; that is that u was a linear controller. Since this has been excluded, it must be that $e_{n+1} = 0$.

A linear relationship $e_1x_1 + e_2x_2 +\ldots+ e_nx_n = 0$ is thus assumed true and it is shown that this implies $e_1 = e_2 =\ldots= e_n = 0$. $e = (e_1,e_2,\ldots,e_n)$ is taken as a row matrix and the relationship is written in matrix form as $ex = 0$. On differentiation this yields $e\dot{x} = 0$. Then by use of equation (1) $eAx + ebu = 0$. Because u is a non-linear controller, it must be that $eb = 0$. Thus, $eAx = 0$ and by differentiation $eA\dot{x} = 0$. This, by equation 1, yields $eA^2x + eAb = 0$. Because u is a non-linear controller, it must also be that $eAb = 0$. Continuing in this manner there results $eb = 0$, $eAb = 0, \ldots eA^{n-1}b = 0$. That is, e is orthogonal to n linearly independent n-dimensional vectors, i.e., e is the zero vector, which was to be proved.

CONCLUSIONS

It has been demonstrated that, given a controllable plant with non-linear controller and given time histories of n functions z_i which depend on the state components x_i through a non-singular linear transformation, it is possible by the method of least squares to establish the matrix of the transformation. The demonstration establishes, for the flexible vehicle problem, the possibility of an "adaptive system" capable of state determination without complete knowledge of mode shapes.

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